Miscellaneous

**Ideal gases**

**Pressure** ($P$) pascal (Pa): $1 \text{ Pa} = 1 \text{ newton per square metre}$.

$$P = \frac{1}{3} \rho \overline{c^2}$$

where $\rho$ is the number density of molecules per cubic metre and $\overline{c^2}$ is the root mean square speed of the molecules. For $n$ moles of gas $PV = nRT$.

**Thermal effects**

**Temperature** $T$ (kelvin) = $t$ (Celsius) + 273.15

Heat $\Delta Q$ supplied to a mass $m$ of a substance with specific heat capacity $c$ results in a temperature rise $\Delta T = \frac{\Delta Q}{mc}$. Atoms within a substance at temperature $T$ typically have energy $\sim k_B T$, where $k_B$ is the Boltzmann constant. In thermal equilibrium, two states 1 and 2 differing in energy by $\Delta E = E_2 - E_1$ have relative populations given by a Boltzmann factor $N_2/N_1 = e^{-\frac{\Delta E}{k_B T}}$.

**Elastic materials**

**Stress** $\sigma = \frac{F}{A} \text{ (Pa)}$ : **Strain** $\varepsilon = \frac{X}{L}$ (dimensionless)

**The Young Modulus** $E = \frac{\sigma}{\varepsilon} \text{ (Pa)}$  

**Strain energy** $\frac{1}{2} k x^2$

TIP: compare this formula with that for energy stored in a capacitor.

**Atomic energy and line spectra**

Electrons in atoms regarded as matter waves.

De Broglie wavelength for a particle of mass $m$ and speed $v$ is $\lambda = \frac{h}{mv}$, where $h$ is the Planck constant. Series of “allowed” energy levels $E_1, E_2$ etc and consequent characteristic spectrum.

**Photoelectric effect**

Photons incident on a surface may cause electrons to be emitted. Maximum kinetic energy of the emitted electrons is determined by the frequency of incident radiation and the substance being illuminated.

**Simple harmonic motion**

Occurs when the force on an object of mass $m$ is directed towards a point $(x = 0)$ and its magnitude is proportional to the distance from that point. $F = -kx$

Oscillation frequency, $f$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Maximum speed = $\omega A$ ($A =$ amplitude)

Maximum acceleration = $\pm \omega^2 A$

Energy of oscillation, $E$

$$E = \frac{1}{2} kA^2 = \frac{1}{2} k v^2 + \frac{1}{2} kx^2$$
Mechanics

**Mechanical quantities**

**Mass** \((m)\) kilogram (kg) **Scalar**
The mass of an object is a measure of its inertia: the difficulty of changing its velocity.*
1 kg is the mass of the international prototype of the kilogram stored at BIPM in Paris.

**Force** \((F)\) newton (N) **Vector**
An unbalanced force causes a mass to accelerate: \(a = \frac{F}{m}\).
1 newton is the force required to accelerate 1 kg at 1 m s\(^{-2}\).
The weight \(W\) of an object is the (attractive) gravitational force acting on it: \(W = mg\).
On the Earth’s surface the gravitational field strength is \(g = 9.8\) N/kg, so 1 kg weighs approximately 9.8 N.

**Energy** \((E)\) joule (J) **Scalar**
1 joule is the energy change when a force of 1 newton acts through 1 metre.
Gravitational potential energy change = weight \(\times\) vertical distance moved = \(mg h\).
Kinetic energy (KE) = \(\frac{1}{2}mv^2\).

**Power** \((P)\) watt (W)
Rate of transferring energy 1 watt = 1 J/s

**Momentum** \((p)\) mass \(\times\) velocity \((kg \, m/s)\) or N s **Vector**
Force = rate of change of momentum: force \(\times\) time (impulse) = momentum change

**Equations of motion** \(v = u + at\) : \(v^2 - u^2 = 2as\) : \(s = ut + \frac{1}{2}at^2\) (for constant acceleration)

**Conservation laws**
Always apply providing the entire system is taken into account:
• **energy is conserved**, but can transfer from one form to another;
• **momentum is conserved**.

**Circular motion**
Assume speed is constant (but velocity changing),
\(\omega = \) angular velocity \((v/r)\) (radian/second), acceleration (toward centre) = \(v^2/r = \omega^2 r\),
\(T = \) period = time for 1 rotation \(T = 2\pi/\omega\)
Electric circuits

**Current and circuits**

**Charge** \((Q)\) coulomb \((C)\)  
1 coulomb is the SI unit of charge.

**Current** \((I)\) ampere \((A)\)  
1 ampere is a current of 1 coulomb per second*.

**Potential difference** \((V)\) volt \((V)\)  
1 volt is the potential difference (PD) between two points when 1 joule of electrical work is done per coulomb moving between those points.

**Power** \((P)\) watt \((W)\)  
Energy dissipated per second = \(IV\).

**Resistance** \((R)\) ohm \((Ω)\)  
1 ohm is one volt per ampere: \(R = V/I\) *

In series:  
\[ R_{\text{Total}} = R_1 + R_2 \]

In parallel:  
\[ \frac{1}{R_{\text{Total}}} = \frac{1}{R_1} + \frac{1}{R_2} \]

**Cells and electromotive force** (EMF)

The EMF \((\varepsilon)\) = the energy supplied to each coulomb by the cell.  
Some energy transferred in external resistance \(R\) and some in internal resistance \(r\).

Electrical work done per coulomb through resistor is \(V = IR\).  
Electrical work done per coulomb through cell \(r\) is \(v = Ir\).

So \(\varepsilon = IR + Ir\).

PD across external resistor \(V = \varepsilon - v\).

**Capacitors**

\(Q\) is the charge displaced from one plate to the other via the circuit.

Capacitance \((C)\) farad \((F)\): the number of coulombs displaced per volt.

\[ C = \frac{Q}{V} \text{ energy stored} = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C} \]

**TIP:** compare with elastic materials.

In series:  
\[ \frac{1}{C_{\text{Total}}} = \frac{1}{C_1} + \frac{1}{C_2} \]

In parallel:  
\[ C_{\text{Total}} = C_1 + C_2 \]

**Parallel plate capacitors**

\(C = \varepsilon_r \varepsilon_0 A/d\) (for parallel plates of area \(A\) separated by distance \(d\) of material with relative permittivity \(\varepsilon_r\)).

**Capacitor discharge through resistor**

**TIP:** compare with radioactive decay.

PD across \(R\):  
\(V = Q/C\), and \(I = V/R\)

Thus \(I = Q/RC\) so \(I\) is proportional to \(Q\)

So rate of loss of \(Q\) (i.e. \(I\)) is proportional to \(Q\)

Therefore \(Q = Q_0 e^{-t/RC}\).

\(RC\) is the time constant = time for \(Q\) to fall to \(1/e\) of original value.  
99% discharge after about \(5RC\) seconds.
### The seven SI base units

The SI (Système International d’Unités) is the standard system of units for scientists worldwide. There are seven base units, from which the other units are derived.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Unit</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>kg</td>
<td>mass</td>
<td>kilogram</td>
<td>The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.</td>
</tr>
<tr>
<td>m</td>
<td>length</td>
<td>metre</td>
<td>The metre is the length of the path travelled by light in vacuum during a time interval of $1/299,792,458$ of a second.</td>
</tr>
<tr>
<td>s</td>
<td>time</td>
<td>second</td>
<td>The second is the duration of $9,192,631,770$ periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium $133$ atom.</td>
</tr>
<tr>
<td>A</td>
<td>electric current</td>
<td>ampere</td>
<td>The ampere is that constant current that, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed $1$ m apart in vacuum, would produce between these conductors a force equal to $2 \times 10^{-7}$ newton per metre of length.</td>
</tr>
<tr>
<td>K</td>
<td>temperature</td>
<td>kelvin</td>
<td>The kelvin, unit of thermodynamic temperature, is the fraction $1/273.16$ of the thermodynamic temperature of the triple point of water.</td>
</tr>
<tr>
<td>cd</td>
<td>luminous intensity</td>
<td>candela</td>
<td>The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540 \times 10^{12}$ hertz and that has a radiant intensity in that direction of $1/683$ watt per steradian.</td>
</tr>
<tr>
<td>mol</td>
<td>amount of substance</td>
<td>mole</td>
<td>The mole is the amount of substance of a system that contains as many elementary entities as there are atoms in $0.012$ kg of carbon 12. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.</td>
</tr>
</tbody>
</table>

### Prefixes

<table>
<thead>
<tr>
<th>Greater than unity</th>
<th>10³</th>
<th>10⁶</th>
<th>10⁹</th>
<th>10¹²</th>
<th>10¹⁵</th>
<th>10¹⁸</th>
<th>10²¹</th>
<th>10²⁴</th>
</tr>
</thead>
<tbody>
<tr>
<td>kilo</td>
<td>Mega</td>
<td>Giga</td>
<td>Tera</td>
<td>Peta</td>
<td>Exa</td>
<td>Zetta</td>
<td>Yotta</td>
<td></td>
</tr>
<tr>
<td>Less than unity</td>
<td>10⁻³</td>
<td>10⁻⁶</td>
<td>10⁻⁹</td>
<td>10⁻¹²</td>
<td>10⁻¹⁵</td>
<td>10⁻¹⁸</td>
<td>10⁻²¹</td>
<td>10⁻²⁴</td>
</tr>
<tr>
<td>milli</td>
<td>µ (micro)</td>
<td>nano</td>
<td>pico</td>
<td>femto</td>
<td>atto</td>
<td>zepto</td>
<td>yocto</td>
<td></td>
</tr>
</tbody>
</table>
## SI derived units

<table>
<thead>
<tr>
<th>Derived quantity</th>
<th>SI derived unit</th>
<th>Base units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceleration</td>
<td>metre per second squared</td>
<td>m s⁻²</td>
</tr>
<tr>
<td>Force</td>
<td>newton (N)</td>
<td>kg m s⁻²</td>
</tr>
<tr>
<td>Work</td>
<td>joule (J)</td>
<td>kg m² s⁻²</td>
</tr>
<tr>
<td>Pressure/strain</td>
<td>pascal (Pa)</td>
<td>kg m⁻¹ s⁻²</td>
</tr>
<tr>
<td>Charge</td>
<td>coulomb (C)</td>
<td>A s</td>
</tr>
<tr>
<td>EMF/potential difference</td>
<td>volt (V)</td>
<td>A⁻¹ kg m² s⁻³</td>
</tr>
<tr>
<td>Electric field strength</td>
<td>newton per coulomb (NC⁻¹) or volt per metre (Vm⁻¹)</td>
<td>A⁻¹ kg m s⁻³</td>
</tr>
<tr>
<td>Capacitance</td>
<td>farad (F)</td>
<td>A² kg⁻¹ m⁻² s⁴</td>
</tr>
<tr>
<td>Magnetic field strength</td>
<td>tesla (T)</td>
<td>A⁻¹ kg s⁻²</td>
</tr>
<tr>
<td>Concentration (of amount of substance)</td>
<td>mole per cubic metre</td>
<td>mol m⁻³</td>
</tr>
</tbody>
</table>

### Useful websites
- National Physical Laboratory outreach: [www.npl.co.uk/educate-explore](http://www.npl.co.uk/educate-explore)
- Institute of Physics: [www.iop.org/careers](http://www.iop.org/careers)
- Your guide to physics on the web: [www.physics.org](http://www.physics.org)
- Physical and chemical constants reference: [www.kayelaby.npl.co.uk](http://www.kayelaby.npl.co.uk)

### Author’s notes
- This is intended as a quick revision guide and not a definitive reference.
- * indicates an equation is correct but not the definition.

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### IOP Institute of Physics

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Radioactivity

Nuclear structure

Atomic (proton) number \( Z \) = number of protons in the nucleus. In a neutral atom this is also the number of electrons in the atom (determines the chemical properties).

Mass (nucleon) number \( A \) = number of protons plus number of neutrons.

The strong nuclear force binds neutrons to neutrons, protons to protons and neutrons to protons. Number of neutrons (\( n \)) is approximately the same as the number of protons (\( p \)).

Isotopes. Atoms with same atomic number \( Z \) (and so chemically similar), but different mass number \( A \). Isotope shown as \( ^A_ZK \), where \( K \) is the chemical symbol. Nuclei have the same number of protons but different numbers of neutrons.

Nuclear decay

Nuclei typically decay by emitting one of two types of particle (\( \alpha \) or \( \beta \)) accompanied by high-frequency electromagnetic radiation known as \( \gamma \) radiation.

<table>
<thead>
<tr>
<th>Alpha (( \alpha )) decay</th>
<th>Beta (( \beta )) minus emission</th>
<th>Gamma (( \gamma )) radiation</th>
</tr>
</thead>
<tbody>
<tr>
<td>An alpha particle (helium nucleus: ( ^4_2\text{He}, 2p + 2n ) is emitted from the nucleus. It is generally followed by a cascade of ( \gamma ) radiation.</td>
<td>A beta particle (electron) is emitted from the nucleus along with an antineutrino. It is generally followed by a cascade of ( \gamma ) radiation.</td>
<td>High-frequency electromagnetic wave (( f &gt; 10^{17} ) Hz).</td>
</tr>
</tbody>
</table>

Decay constant

Probability of decay in a fixed time: \( \lambda = -\frac{1}{N} \left[ \frac{dN}{dt} \right] \)

When some have decayed, fewer remain so the rate of decay (the activity) falls. \( N = N_0 e^{-\lambda t} \)

TIP: compare with decay of charge on a capacitor.

Half-life (\( T_{\frac{1}{2}} \))

Time for half to decay \( T_{\frac{1}{2}} = \ln(2)/\lambda = 0.69/\lambda \).

Radioactivity quantities and units

Activity becquerel (Bq) is 1 disintegration per second.

Absorbed dose gray (Gy) is the dose when 1 joule is absorbed by 1 kg of tissue.

Dose equivalent sievert (Sv) is related to the biological harm caused by the absorbed dose.*

Binding energy

If protons and neutrons are bound into a nucleus, nuclear mass is \( \Delta m \) less than the sum of the masses of the constituent protons and neutrons.

Binding energy = \( \Delta mc^2 \), where \( c \) is the speed of light in a vacuum.
Fields

**Fields due to an isolated spherical charge or mass**

Inverse square law of force due to an isolated charge \((Q)\) or mass \((M)\),

\[
F = \frac{kQq}{r^2} \quad \text{(where } k = \frac{1}{4\pi\varepsilon_0} \text{ in a vacuum)} : \quad F = -\frac{GMm}{r^2} \quad \text{(where } G \text{ is gravitational constant)}.
\]

**Field strength** Vector. Examples are force per unit charge \(E\), or force per unit mass \(g\).

Force on 1 coulomb \(F = \frac{kQ}{r^2}\) : Force on 1 kilogram \(F = \frac{GM}{r^2}\).

For fields within a substance of relative permittivity \(\varepsilon_r\) electric fields are reduced by factor \(\varepsilon_r\). There is no known type of “gravitational permittivity”.

**Field strength = negative potential gradient** \(= -\frac{dV}{dr}\).

**Field potential**

**Field potential** \((V)\) Scalar. Potential energy of unit electric charge (or unit mass).

Energy required to bring unit electric charge (or mass) from infinity to the point in question.

- **Electrical** (repulsive force for positive \(Q\)): \(V_{\text{elec}} = \frac{kQ}{r}\)
- **Gravitation** (attractive force for positive \(M\)): \(V_{\text{grav}} = -\frac{GM}{r}\).

**Potential energy** of charge \(q\) (or mass \(m\)) in the field: \(qV_{\text{elec}}\) (or \(mV_{\text{grav}}\)).

**Uniform fields.** Field lines are parallel; field strength is given by the negative of potential gradient \(E = -V/d\).

**Magnetic fields**

**Magnetic field strength** \((B)\) tesla \((T)\) Vector. 1 tesla is the magnetic field strength that gives rise to a force of 1 newton per metre on a wire carrying 1 ampere.

Density of field lines in diagrams is proportional to field strength.

**Magnetic flux** \((\Phi)\) weber \((\text{Wb})\)

Through an area \(A : \Phi = BA\) (field lines perpendicular to \(A\)).

**Forces in a magnetic field**

- Force on a wire length \(l\) carrying current \(I\) perpendicular to field \(B\) has magnitude \(F = BIl\), which acts perpendicular to \(B\) and \(I\) as given by a left-hand rule.
- Force on a charge \(q\) travelling with speed \(v\) perpendicular to magnetic field \(B\) has magnitude \(F = Bqv\), which acts perpendicular to \(B\) and \(v\) as given by a left-hand rule. Charge moves in arc of circle of radius \(r = mv/qB\).

**Induced EMF in a magnetic field**

For a loop of wire threaded by flux \(\Phi\), \(\varepsilon = -\frac{d\Phi}{dt}\). For a coil of \(N\) loops each with flux \(\Phi\), \(\varepsilon = -N\frac{d\Phi}{dt}\).
Waves

Energy transfer by waves
Transfer of energy without the transfer of matter.

Speed of waves
For both transverse and longitudinal \( v = f \lambda \):
\( v \) = speed; \( f \) = frequency; \( \lambda \) = wavelength

Interference
Diffraction pattern from single slit
Diffraction results from interruption of part of the wavefront. An explanation can be given using Huygen’s construction.
Electrons and other particles can be diffracted to show their wave properties.
For first minimum \( \sin \theta = \frac{\lambda}{b} \)
For small angles \( \sin \theta \approx \theta \) (rads)

Double slit
Assume that waves at each slit are coherent and in-phase.
For constructive interference on the screen, the path difference = \( n \lambda \).
\( \sin \theta = \frac{n\lambda}{d} \); \( \frac{\lambda}{d} = \frac{s}{L} \) (for small angles)

Diffraction grating (multiple slits)
Different wavelengths produce constructive interference at different angles given by \( n \lambda = d \sin \theta \).
Same formula as two slits, but grating forms narrow distinct lines. \( d \) usually small, so \( \theta \) large.

Electromagnetic radiation
Speed of light in a vacuum: \( c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \)
Refractive index measures the reduction in the speed of light within a medium.
A refractive index of 1.5 means light travels at 1/1.5 times the speed of light in a vacuum.
Energy of a photon is \( E = hf \)
(\( h \) is the Planck constant).
<table>
<thead>
<tr>
<th>Data</th>
<th>Unit</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceleration of free fall (in UK)</td>
<td>g</td>
<td>9.81</td>
<td>m s⁻²</td>
</tr>
<tr>
<td>Gravitational field strength (in UK)</td>
<td>g</td>
<td>9.81</td>
<td>N kg⁻¹</td>
</tr>
<tr>
<td>Gravitational constant</td>
<td>G</td>
<td>6.67 × 10⁻¹¹</td>
<td>N m kg⁻²</td>
</tr>
<tr>
<td>Electric force constant ( k = \frac{1}{4\pi\varepsilon_0} )</td>
<td></td>
<td>8.98 × 10⁹</td>
<td>N m² C⁻¹</td>
</tr>
<tr>
<td>Speed of light in a vacuum</td>
<td>c</td>
<td>3.00 × 10⁸</td>
<td>m s⁻¹</td>
</tr>
<tr>
<td>Permeability of free space</td>
<td>( \mu_0 )</td>
<td>4(\pi) × 10⁻⁷</td>
<td>N A⁻²</td>
</tr>
<tr>
<td>Permittivity of free space</td>
<td>( \varepsilon_0 )</td>
<td>8.85 × 10⁻¹²</td>
<td>F m⁻¹</td>
</tr>
<tr>
<td>Planck constant</td>
<td>h</td>
<td>6.63 × 10⁻³⁴</td>
<td>J s</td>
</tr>
<tr>
<td>Elementary electron charge</td>
<td>e</td>
<td>$-1.60 \times 10^{-19}$</td>
<td>C</td>
</tr>
<tr>
<td>Electron rest mass</td>
<td>m</td>
<td>$9.11 \times 10^{-31}$</td>
<td>kg</td>
</tr>
<tr>
<td>Electronvolt</td>
<td>eV</td>
<td>$1.60 \times 10^{-19}$</td>
<td>J</td>
</tr>
<tr>
<td>Unified atomic mass constant</td>
<td>u</td>
<td>$1.66 \times 10^{-27}$</td>
<td>kg</td>
</tr>
<tr>
<td>Proton rest mass</td>
<td>( m_p )</td>
<td>$1.673 \times 10^{-27}$</td>
<td>kg</td>
</tr>
<tr>
<td>Neutron rest mass</td>
<td>( m_n )</td>
<td>$1.675 \times 10^{-27}$</td>
<td>kg</td>
</tr>
<tr>
<td>Molar gas constant</td>
<td>R</td>
<td>8.31</td>
<td>J K⁻¹ mol⁻¹</td>
</tr>
<tr>
<td>Boltzmann constant</td>
<td>( k_b )</td>
<td>$1.38 \times 10^{-23}$</td>
<td>J K⁻¹</td>
</tr>
<tr>
<td>Avogadro constant</td>
<td>( N_A )</td>
<td>$6.02 \times 10^{23}$</td>
<td>mol⁻¹</td>
</tr>
<tr>
<td>Standard temperature and pressure</td>
<td>STP</td>
<td>273.15 K and $1.01 \times 10^5$ Pa</td>
<td></td>
</tr>
<tr>
<td>Molar volume at STP</td>
<td>( V_m )</td>
<td>$22.4 \times 10^{-3}$</td>
<td>m³ mol⁻¹</td>
</tr>
</tbody>
</table>